2.6 Trig functions and limits

2014-09-25 day 22

There is a semi-important technique To know in 26.

Remember: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

So... $\lim_{2x \to 0} \frac{\sin 2x}{2x} = 1$ and $\lim_{3x \to 0} \frac{\sin 3x}{3x} = 1$ and so on

but when is 2x headed to?

$2x \to 0$ iff $x \to 0$

So.... $\lim_{x \to 0} \frac{\sin 2x}{2x} = 1$, $\lim_{x \to 0} \frac{\sin 3x}{3x} = 1$, and so on...

2.6 Trig functions and limits

2014-09-25 day 22

how do we use these facts?

$$\lim_{x\to 0} \frac{2\sin(2x)}{2x} = 2\lim_{x\to 0} \frac{\sin(2x)}{2x}$$

$$= 1$$

$$= 2(1) = 2$$

$$\lim_{x\to 0} cf(x) = c\lim_{x\to 0} f(x)$$

Caveat: x is in RADIANS

2.6 Trig functions and limits

2014-09-25 day 22

$y = \frac{\sin(x)}{x} \hookrightarrow 1$ $y = \frac{\sin(2x)}{x} \hookrightarrow 2$

$y = \frac{\sin(3x)}{2x} \hookrightarrow \frac{3}{2}$

"÷(2x)"

2.6 Trig functions and limits

2014-09-25 day 22

$$\lim_{x\to 0}\frac{\sin(3x)}{2x} = \frac{1}{2}\lim_{x\to 0}\frac{\sin(3x)}{x}$$

$$= \frac{1}{2}\lim_{x\to 0}3\frac{\sin(3x)}{3x} = \frac{3}{2}\lim_{x\to 0}\frac{\sin(3x)}{3x} = \frac{3}{2}$$

2.6 Trig functions and limits 2014-09-25 day 22

$\lim_{x\to a} c\,f(x) = c\lim_{x\to a} f(x)$

$$\lim_{x\to 0}\frac{\sin(2x)}{3x} = \lim_{x\to 0}\left(\frac{2}{3}\right)\frac{\sin(2x)}{2x} = \frac{2}{3}\lim_{x\to 0}\frac{\sin(2x)}{2x}$$

$\lim_{x\to 0}\frac{\sin(2x)}{2x} = 1$

$$= \frac{2}{3}(1) = \frac{2}{3}$$

2.6 Trig functions and limits

2014-09-25 day 22

$$\lim_{x\to 0} \frac{\sin(3x)}{\sin(7x)} = \lim_{x\to 0} \frac{3}{7}\cdot\frac{(7x)}{\sin(7x)}\cdot\frac{\sin(3x)}{(3x)}$$

$$= \frac{3}{7}\lim_{x\to 0}\left(\frac{7x}{\sin(7x)}\right)\left(\frac{\sin(3x)}{3x}\right)$$

$$= \frac{3}{7}(1)(1) = \frac{3}{7}$$

$$\lim_{x\to 0}\frac{x}{\sin x} = \frac{1}{\lim_{x\to 0}\frac{\sin x}{x}} = \frac{1}{(1)} = 1$$

$$\frac{\sin(3x)}{(3x)} = \sin \quad \lim_{x\to 0}$$

2.6 Trig functions and limits

2014-09-25 day 22

$$\lim_{x\to 0}\frac{\tan(4x)}{\sin(7x)} = \lim_{x\to 0}\left(\frac{4}{7}\right)\frac{\sin(4x)}{\cos(4x)}\frac{(7x)}{(4x)}\frac{1}{\sin(7x)} = \frac{4}{7}$$

$$\lim_{x\to 0}\frac{\left(\frac{\sin(4x)}{\cos(4x)}\right)\left(\frac{1}{\sin(7x)}\right)}{\left(\frac{\sin(7x)}{1}\right)\left(\frac{1}{\sin(7x)}\right)} = \lim_{x\to 0}\frac{\frac{\sin(4x)}{\cos(4x)(\sin(7x))}}{\frac{(\sin(7x))}{\sin(7x)}}$$

$$\frac{\left(\frac{a}{b}\right)\left(\frac{1}{c}\right)}{\frac{c}{1}\left(\frac{1}{c}\right)} = \frac{a}{bc}$$

$$= \lim_{x\to 0}\frac{\sin(4x)}{\cos(4x)\sin(7x)}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{1}\cdot\frac{c}{b} = \frac{ac}{b}$$

$$\lim_{x\to 0}\frac{\sin(4x)}{\cos(4x)\sin(7x)} = \frac{4}{7}\lim_{x\to 0}\frac{\sin(4x)}{4x}\cdot\frac{1}{\cos(4x)}\cdot\frac{7x}{\sin(7x)}$$

$$\lim_{x\to 0}\left(\frac{4}{7}\right)\left(\frac{\sin(4x)}{4x}\right)\left(\frac{1}{\cos(4x)}\right)\left(\frac{7x}{\sin(7x)}\right)$$

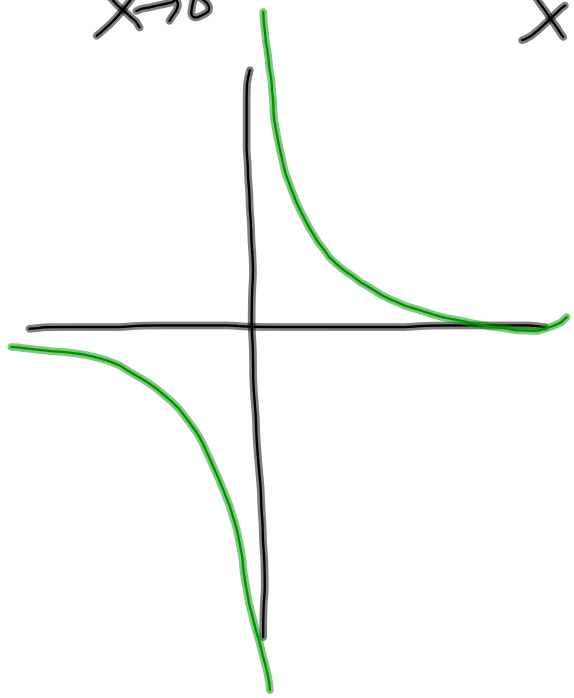
2.6 Trig functions and limits

2014-09-25 day 22

18) $\lim_{x \to 0^+} \frac{\sin x}{x^2} = \lim_{x \to 0^+} \left(\frac{\sin x}{x}\right)\left(\frac{1}{x}\right) = +\infty$

$\frac{\sin x}{x} \to 1$

$\frac{1}{x} \to +\infty$ DNE



2.6 Trig functions and limits

2014-09-25 day 22

21) $\lim_{x\to 0^+} \frac{\sin(x)}{5\sqrt{x}} = \lim_{x\to 0^+} \left(\frac{x}{5\sqrt{x}}\right)\left(\frac{\sin x}{x}\right)$

$\frac{x^1}{x^{1/2}} = x^{1-1/2} = x^{1/2} = \sqrt{x}$

$\lim_{x\to 0^+} \left(\frac{\sqrt{x}}{5}\right)\left(\frac{\sin x}{x}\right) = 0$

$\Downarrow$

$\lim_{x\to 0} \left(\frac{\sqrt{x}}{5}\right) \cdot \lim_{x\to 0}\left(\frac{\sin x}{x}\right)$